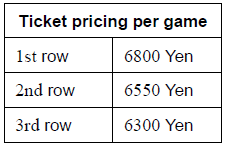
# 1.2 arithmetic sequences and series\_P\_1

**1a.** *[1 mark]*

The Osaka Tigers basketball team play in a multilevel stadium.



The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.



Write down the value of the common difference, 

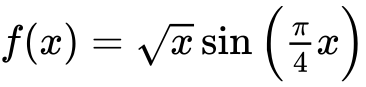
**1b.** *[2 marks]*

Calculate the price of a ticket in the 16th row.

**1c.** *[3 marks]*

Find the total cost of buying 2 tickets in each of the first 16 rows.

**2a.** *[5 marks]*

Consider  and  for  ≥ 0. The first time the graphs of  and  intersect is at .

Find the **two** smallest non-zero values of  for which .

**2b.** *[4 marks]*

The set of all non-zero values that satisfy  can be described as an arithmetic sequence,  where  ≥ 1.

Find the value of  and of .

**2c.** *[4 marks]*

At point P, the graphs of  and  intersect for the 21st time. Find the coordinates of P.

**3.** *[4 marks]*

In an arithmetic sequence, the sum of the 3rd and 8th terms is 1.

Given that the sum of the first seven terms is 35, determine the first term and the common difference.

**4a.** *[2 marks]*

In an arithmetic sequence, *u* = −5 and *d* = 3.

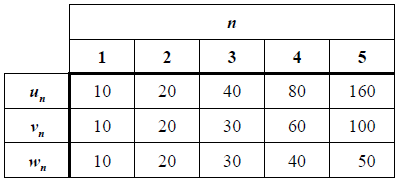
Find *u*.

**4b.** *[4 marks]*

Find the value of *n* for which *u* = 67.

**5a.** *[1 mark]*

The table shows the first five terms of three sequences: *u* , *v* and *w*.



State which sequence is arithmetic.

**5b.** *[1 mark]*

State which sequence is geometric.

**5c.** *[2 marks]*

Find the exact value of the 11th term of the geometric sequence.

**5d.** *[2 marks]*

Find the sum of the first 20 terms of the arithmetic sequence.

**6a.** *[2 marks]*

Sergei is training to be a weightlifter. Each day he trains at the local gym by lifting a metal bar that has heavy weights attached. He carries out successive lifts. After each lift, the same amount of weight is **added** to the bar to increase the weight to be lifted.

The weights of each of Sergei’s lifts form an arithmetic sequence.

Sergei’s friend, Yuri, records the weight of each lift. Unfortunately, last Monday, Yuri misplaced all but two of the recordings of Sergei’s lifts.

On that day, Sergei lifted 21 kg on the third lift and 46 kg on the eighth lift.

For that day find how much weight was added after each lift.



**6b.** *[2 marks]*

For that day find the weight of Sergei’s first lift.



**6c.** *[2 marks]*

On that day, Sergei made 12 successive lifts. Find the total combined weight of these lifts.



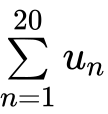
**7a.** *[2 marks]*

An arithmetic sequence has  and , where  and .

Show that .

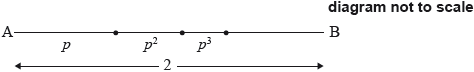


**7b.** *[6 marks]*

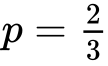
Let  and . Find the value of .

**8.** *[5 marks]*

The following diagram shows [AB], with length 2 cm. The line is divided into an infinite number of line segments. The diagram shows the first three segments.



The length of the line segments are , where .

Show that .

**9a.** *[2 marks]*

In an arithmetic sequence, the first term is 8 and the second term is 5.

Find the common difference.

**9b.** *[2 marks]*

Find the tenth term.

**9c.** *[2 marks]*

Find the sum of the first ten terms.

**10a.** *[3 marks]*

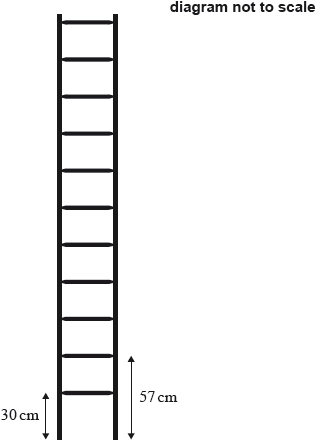
The company Snakezen’s Ladders makes ladders of different lengths. All the ladders that the company makes have the same design such that:

the first rung is 30 cm from the base of the ladder,

the second rung is 57 cm from the base of the ladder,

the distance between the first and second rung is equal to the distance between all adjacent rungs on the ladder.

The ladder in the diagram was made by this company and has eleven equally spaced rungs.



Find the distance from the base of this ladder to the top rung.



**10b.** *[3 marks]*

The company also makes a ladder that is 1050 cm long.

Find the maximum number of rungs in this 1050 cm long ladder.



**11a.** *[2 marks]*

In an arithmetic sequence, the first term is 3 and the second term is 7.

Find the common difference.

**11b.** *[2 marks]*

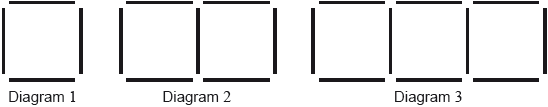
Find the tenth term.

**11c.** *[2 marks]*

Find the sum of the first ten terms of the sequence.

**12a.** *[3 marks]*

Tomás is playing with sticks and he forms the first three diagrams of a pattern. These diagrams are shown below.



Tomás continues forming diagrams following this pattern.

Diagram  is formed with 52 sticks. Find the value of .

**12b.** *[3 marks]*

Tomás forms a total of 24 diagrams.

Find the total number of sticks used by Tomás for all 24 diagrams.

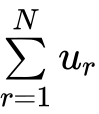
**13a.** *[4 marks]*

An arithmetic sequence  has  and common difference . Given that  and  are the first three terms of a geometric sequence

find the value of .

**13b.** *[3 marks]*

Given that 

determine the value of .

**14a.** *[4 marks]*

The 1st, 4th and 8th terms of an arithmetic sequence, with common difference , , are the first three terms of a geometric sequence, with common ratio . Given that the 1st term of both sequences is 9 find

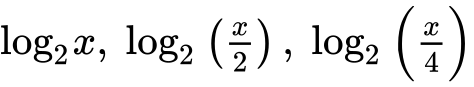
the value of ;

**14b.** *[1 mark]*

the value of ;

**15a.** *[4 marks]*

The first three terms of an arithmetic sequence, in order, are

, where .

Find , giving your answer as an integer.

**15b.** *[2 marks]*

Let  be the sum of the first 12 terms of the arithmetic sequence.

Show that .

**16a.** *[3 marks]*

A comet orbits the Sun and is seen from Earth every 37 years. The comet was first seen from Earth in the year 1064.

Find the year in which the comet was seen from Earth for the fifth time.

**16b.** *[3 marks]*

Determine how many times the comet has been seen from Earth up to the year 2014.



**17a.** *[3 marks]*

Consider the following sequence of figures.

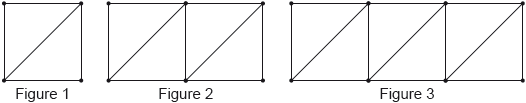


Figure 1 contains 5 line segments.

Given that Figure  contains 801 line segments, show that .

**17b.** *[3 marks]*

Find the total number of line segments in the first 200 figures.

**18a.** *[2 marks]*

One of the locations in the  Olympic Games is an amphitheatre. The number of seats in the first row of the amphitheatre,  , is . The number of seats in each subsequent row forms an arithmetic sequence. The number of seats in the sixth row,  , is .

Calculate the value of the common difference, .

**18b.** *[2 marks]*

There are  rows in the amphitheatre.

Find the **total** number of seats in the amphitheatre.



**18c.** *[2 marks]*

Anisha visits the amphitheatre. She estimates that the amphitheatre has  seats.

Calculate the percentage error in Anisha’s estimate.



**19.** *[6 marks]*

The fifth term of an arithmetic sequence is equal to 6 and the sum of the first 12 terms is 45.

Find the first term and the common difference.

**20.** *[6 marks]*

An arithmetic sequence has the first term  and a common difference .

The 13th term in the sequence is . Find the value of .



**21a.** *[3 marks]*

The second term of an arithmetic sequence is 30. The fifth term is 90.

Calculate

(i)     the common difference of the sequence;

(ii)     the first term of the sequence.

**21b.** *[3 marks]*

The first, second and fifth terms of this arithmetic sequence are the first three terms of a geometric sequence.

Calculate the seventh term of the **geometric** sequence.

**22.** *[4 marks]*

Let , be an arithmetic sequence with first term equal to  and common difference of , where . Let another sequence , be defined by .

(i)     Show that  is a constant.

(ii)     Write down the first term of the sequence .

(iii)     Write down a formula for  in terms of ,  and .

**23a.** *[2 marks]*

In an arithmetic sequence, the first term is  and the second term is .

Find the common difference.

**23b.** *[2 marks]*

Find the eighth term.

**23c.** *[2 marks]*

Find the sum of the first eight terms of the sequence.

**24a.** *[2 marks]*

The number of apartments in a housing development has been increasing by a constant amount every year.

At the end of the first year the number of apartments was 150, and at the end of the sixth year the number of apartments was 600.

The number of apartments, , can be determined by the equation , where is the time, in years.

Find the value of .

**24b.** *[1 mark]*

State what  represents **in this context**.

**24c.** *[2 marks]*

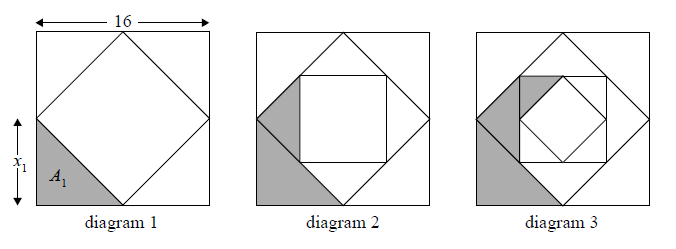
Find the value of .

**24d.** *[1 mark]*

State what  represents **in this context**.

**25a.** *[4 marks]*

The sides of a square are 16 cm in length. The midpoints of the sides of this square are joined to form a new square and four triangles (diagram 1). The process is repeated twice, as shown in diagrams 2 and 3.



Let  denote the length of one of the equal sides of each new triangle.

Let  denote the area of each new triangle.

The following table gives the values of  and , for . **Copy** and complete the table. *(Do* ***not*** *write on this page.)*

 1 2 3  8   4  32 16

**25b.** *[4 marks]*

The process described above is repeated. Find .

**26a.** *[2 marks]*

In an arithmetic sequence, the third term is 10 and the fifth term is 16.

Find the common difference.

**26b.** *[2 marks]*

Find the first term.

**26c.** *[3 marks]*

Find the sum of the first 20 terms of the sequence.

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